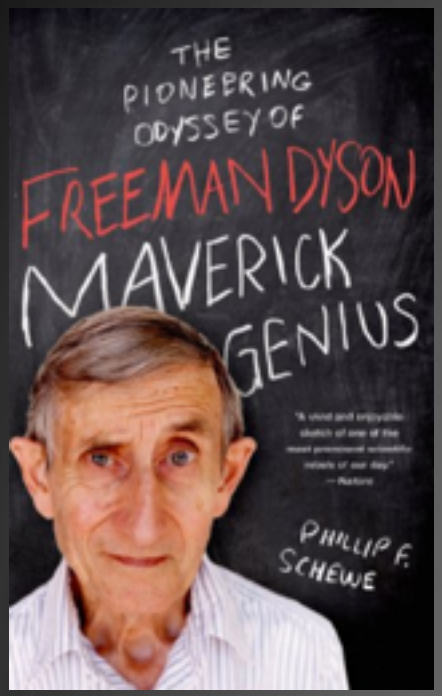
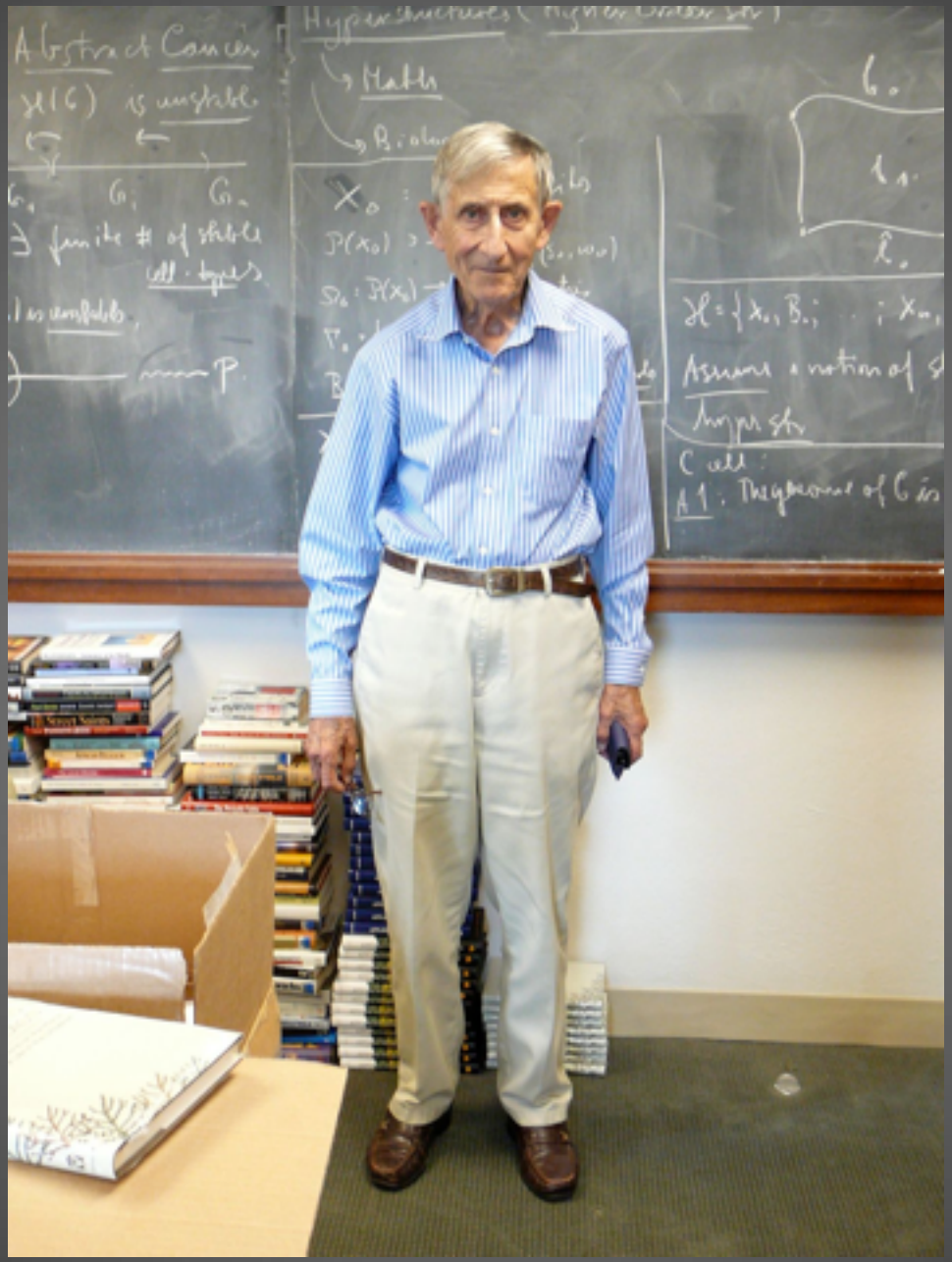
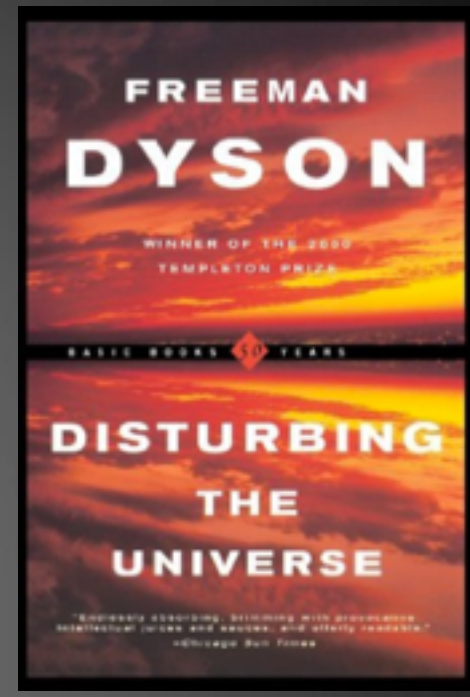
Mathematical Platonism As a Necessity of Reason

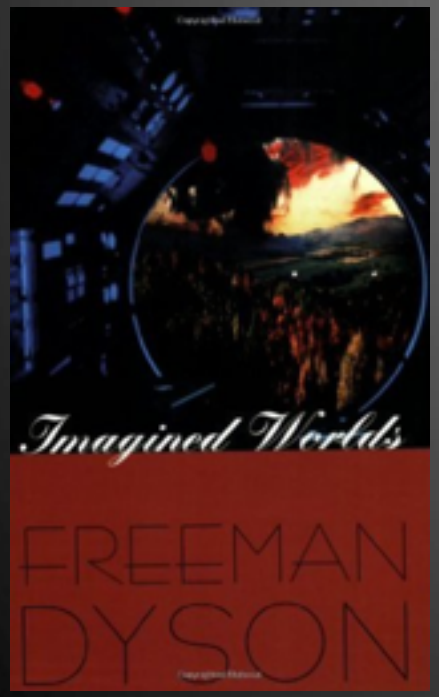
Alexey Burov, Fermilab, Dec 15, 2016

Happy Birthday, Dear Freeman!



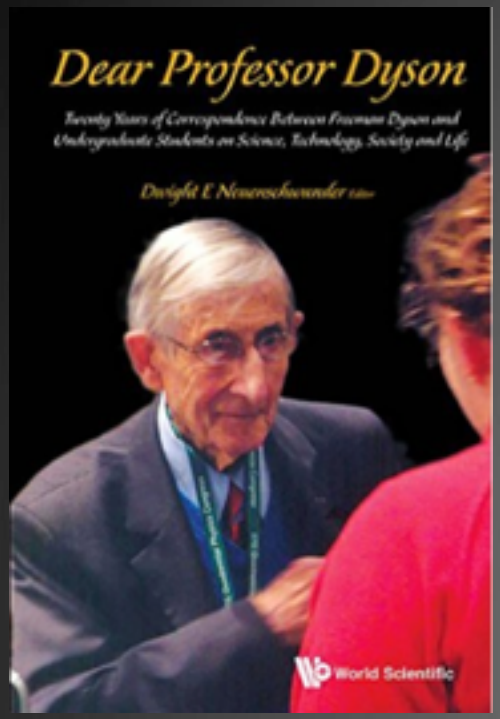


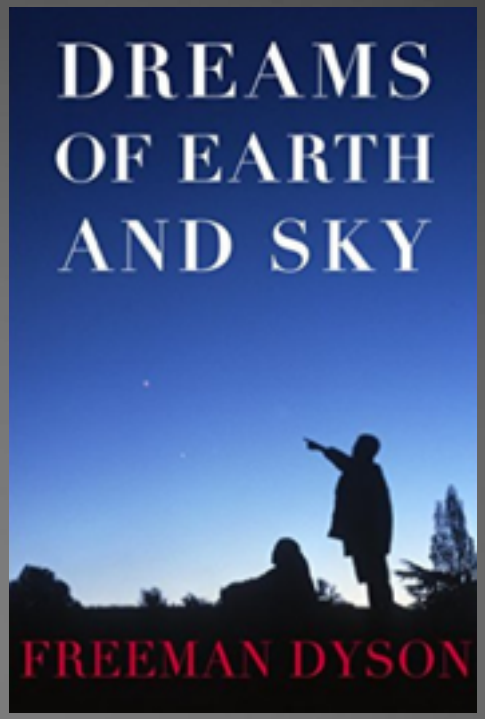


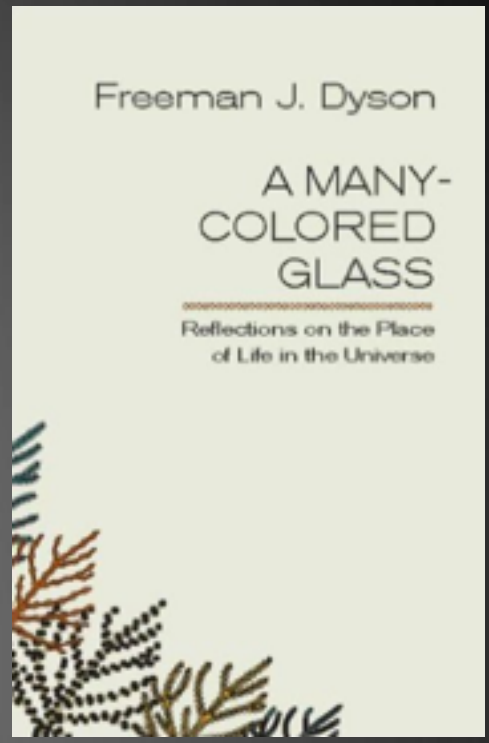


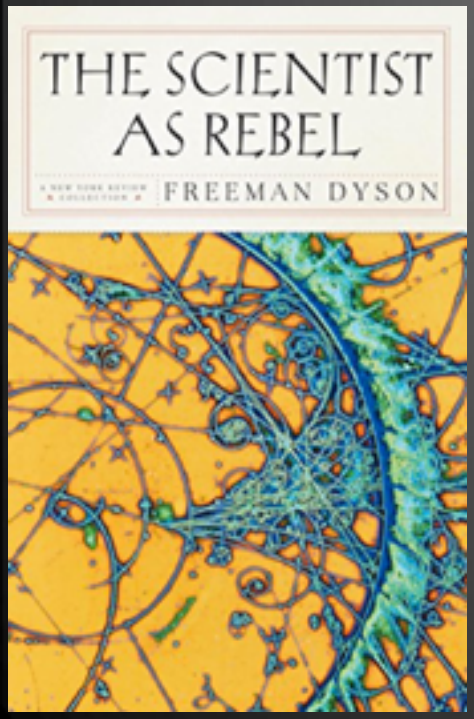
Born：Dec 15, 1923

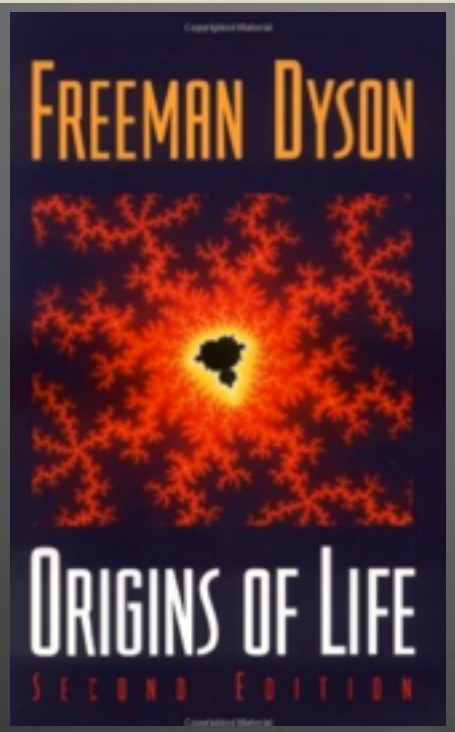
Freeman Dyson, Born Dec. 15, 1923

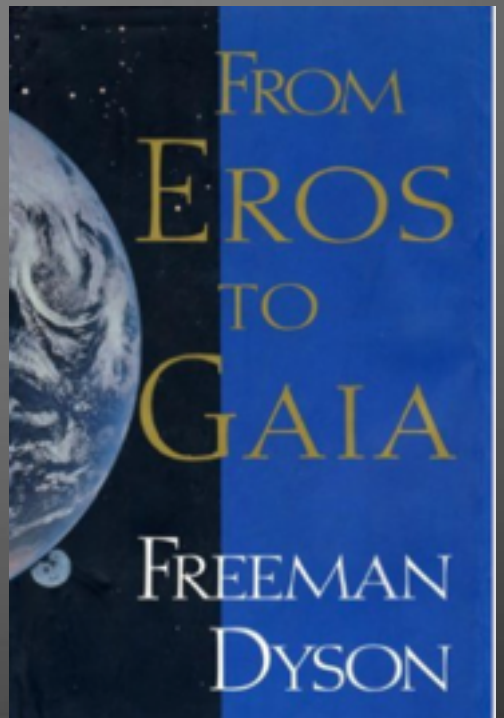


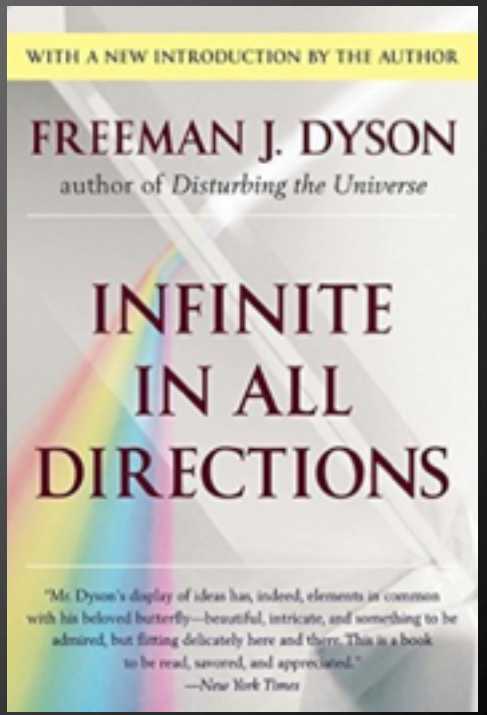












Freeman Dyson, Ideas Roadshow (2014)

Howard Burton: You don't have any Platonist dispositions, do you?

Freeman: Oh yes. If by Platonism you mean the belief that the mathematical world is real, then yes.

Howard: You are a Platonist?

Freeman: Yes. Certainly. I think that's pretty obvious.

Howard: Have you always been a Platonist?

Freeman: In the sense of believing that the mathematical world is real. But of course mathematics has acquired a huge freedom which it didn't have in the old days. Now there is not just one mathematics; there are many.

Howard: But that's neither here nor there with respect to the Platonist argument. A Platonist would say we're just discovering new stuff that's always been there. And as a diehard Platonist, how would you respond to the question of how it's possible that we gain access to this metaphysical world?

Freeman: It's a miracle. It's just a plain miracle.

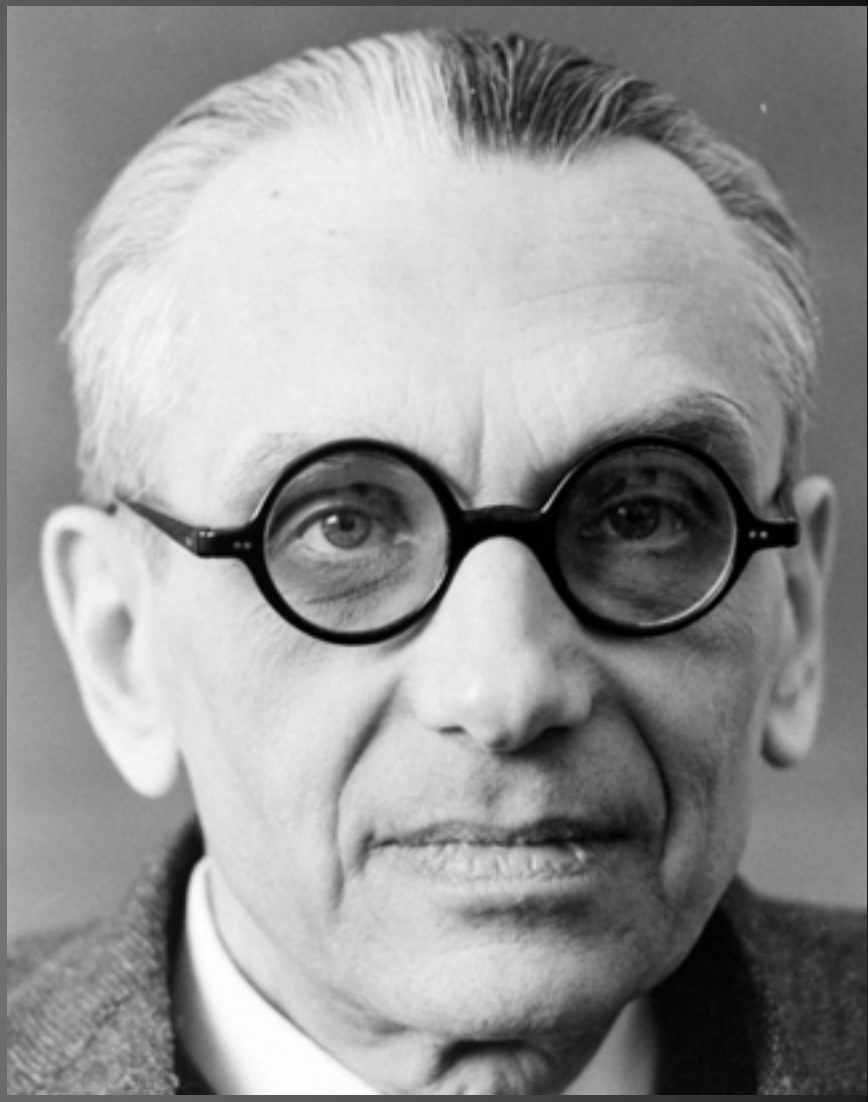
Howard: That's a good answer.

K. Gödel on Platonism

I am under the impression that… the Platonistic view is the only one tenable. Thereby I mean the view that mathematics describes a non-sensual reality, which exists independently both of the acts and the dispositions of the human mind and is only perceived, and probably perceived very incompletely, by the human mind.

There exists, unless I am mistaken, an entire world consisting of the totality of mathematical truths, which is accessible to us only through our intelligence, just as there exists the world of physical realities; each one is independent of us, both of them divinely created.

K. Gödel, 1951, in J. Polkinghorne, Meaning in Mathematics (p. 148). OUP Oxford. Kindle Edition.



1906-1978

G.H. Hardy, "A Mathematician's Apology" (1940)

I believe that mathematical reality lies outside us, that our function is to discover or observe it, and that the theorems which we prove, and which we describe grandiloquently as our 'creations', are simply our notes of our observations. This view has been held, in one form or another, by many philosophers of high reputation from Plato onwards, and I shall use the language which is natural to a man who holds it.

The play of Shakespeare is independent of the pages on which it is printed, and 'pure geometries' are independent of lecture rooms, or of any other detail of the physical world.

317 is a prime, not because we think so, or because our minds are shaped in one way rather than another, but because it is so, because mathematical reality is built that way.

1877–1947

B. Russell (1903); E. Wigner (1960)

Mathematics, rightly viewed, possesses not only truth, but supreme beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show. The true spirit of delight, the exaltation, the sense of being more than Man, which is the touchstone of the highest excellence, is to be found in mathematics as surely as in poetry.

(The Study of Mathematics)

This Russell's quotation stays as an epigraph to E. Wigner's "Unreasonable Effectiveness of Mathematics in Natural Sciences" (1960).

Henri Poincare (1854-1912)

"The sole objective reality consists in the *relations* of things *whence* results the universal harmony. Doubtless these relations, this harmony, could not be conceived outside of a mind which conceives them. But they are nevertheless objective because they are, will become, or will remain, common to all thinking beings."

1854-1912

"If nature were not beautiful it would not be worth knowing, and life would not be worth living. I am not speaking, of course, of the beauty which strikes the senses, of the beauty of qualities and appearances. I am far from despising this, but it has nothing to do with science. What I mean is *that* more intimate beauty which comes from the harmonious order of its parts, and which a pure intelligence can grasp.

"Logic teaches us that on such and such a road we are sure of not meeting an obstacle; it does not tell us which is the road that leads to the desired end. For this, it is necessary to see the end from afar, and the faculty which teaches us to see is *intuition*. Without it, the geometrician would be like a writer well up in grammar but destitute of ideas."

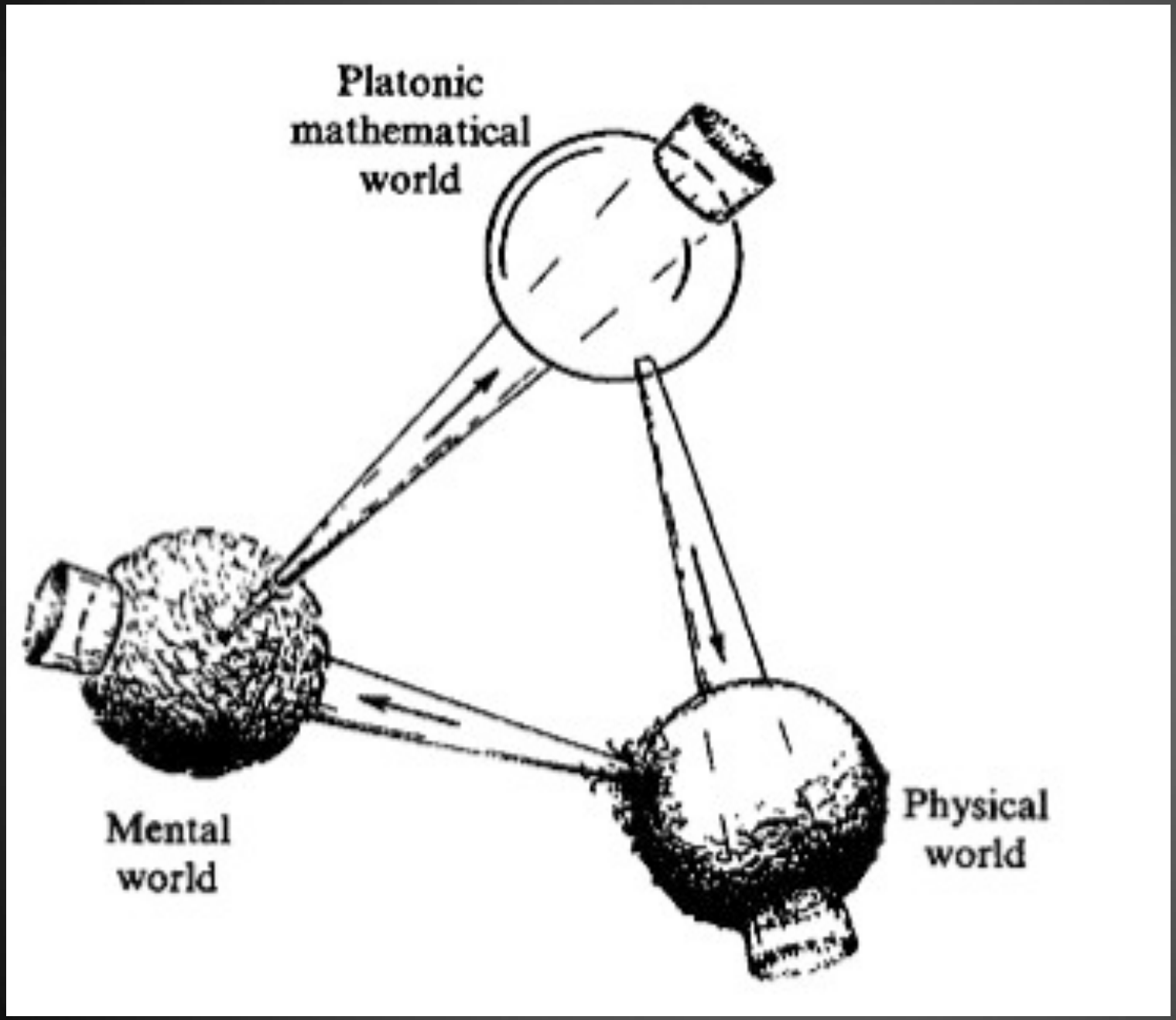
Roger Penrose

"... the concurrence between Nature and *sophisticated beautiful* mathematics is something that is "out there" and has been so since times far earlier than the dawn of humanity, or of any other conscious entities that could have inhabited the universe as we know it."

in J. Polkinghorne, ed., *Meaning in Mathematics*

Born: Aug 8, 1931

Three Worlds, Three Mysteries of Sir Roger



Born: Aug 8, 1931

J. Polkinghorne on Platonism

Another argument in favour of the reality of the physical world is its character of quite often proving surprising, with properties contrary to prior 'reasonable' expectation. Quantum theory is the prime exemplar of this. This resistance to prior expectation is persuasive that we are encountering an independent reality standing over against us. I suppose that the nineteenth-century discovery of non-Euclidean geometries would be a mathematical analogy... Think of the astonishing fruitfulness of the idea of complex numbers. Defenses of realist interpretations in both physics and mathematics have to be subtle and delicate and it seems to me that the two disciplines are cousins under the skin in this respect.

J. Polkinghorne, Meaning in Mathematics (p. 95). OUP Oxford. Kindle Edition.

Born: Oct 16, 1930

Why Is Mathematical World Real?

[It is] the incontestable fact that almost all mathematicians who successfully prove theorems feel as though they are making discoveries.

Sir W. Timothy Gowers, FRS, Fields medalist (born 1963).

"...it would be quite odd to say that before 1982, the Monster group did not exist. If this were the right thing to say, then when Griess first asked himself the question, 'Does the Monster exist?' the answer should have been obvious: 'Not yet, but maybe someday.' But in fact no one speaks of mathematical objects in this way."

G. Rosen (born 1962), Stuart Professor of Philosophy and formerly Chair of the Council of Humanities at Princeton University

Why Is Mathematical World Real?

It has all the features of objective reality:

Its truths are looked for and discovered as independent of human individualities, of cultures, of our biology, of time and place and even of the laws of nature: in whatever world, 317 is prime. Thus, its reality is more universal than anything else.

To communicate with extraterrestrial thinking creatures, mathematical truths are the most reasonable to be tried first.

Math truths are discovered, experimentally tested, and sometimes amaze us. Some of them are 'unreasonably effective' already in mathematics.

Mathematical world is atemporal; its truths do not change with time; they are unshakable.

The Universe is constituted by very special, both elegant and anthropic, mathematical forms; mathematics is "unreasonably effective" in physics.

Can Math Platonic World be just a Babylon Library?

Math is developed for two reasons:

1) for itself;

2) for applications.

Main Math discoveries, from the ancient time, all happened as the 1st case.

Math was driven by a very special values, where necessity of Reason was blended with a special esthetics and religious meaning.

The idea of 'math democracy' (Tegmark) is incompatible with the value of Math. It is also incompatible with Physics ("Genesis of a Pythagorean Universe").

The very existence of Math is due to its special super-human esthetics: formal systems are fundamentally unequal, dull ones are out of the discourse neither in pure Math, nor in the search of laws of nature: 'unreasonable effectiveness of Math' in the fundamental Physics 'is a wonderful gift which we neither understand nor deserve'.

Math opens deeply only to those who feel and highly appreciate its aristocratic esthetics.

Four Existences of Math

1. Math is a human language; as such it belongs to culture. In that sense, imaginary numbers did not yet exist, say, at XV century.

2. As a way of thinking, it belongs to the mental world of a subject. In that sense, in 1545 Cardano constructed rules for the imaginary unit.

3. Words of language point to some reality; math as a language describes the math platonic world. In that sense, complex numbers constitute reality which properties are independent of our specific features, are strange, surprising and 'unreasonably effective'.

4. Structure of the physical world is constituted by special elegant math forms.

Math can be defined as the player, common for all the four fields.

Can Math be just an idealization?

Naturalism denies reality of Math World, claiming it is nothing but human idealization of the material world, which is the only reality. This view does not seem reasonable:

We consider math objects, prove their features independently of their role in the physical world. Math objects have their own connectivity and relations, generalizations and deepening, independently of their roles in physical reality.

Many math entities were developed purely mathematically, before any of their roles in physics were seen, like imaginary numbers or non-euclidian geometry.

A belief that Math is just an idealization of Physical world is not harmless from a pure practical point of view: it hindered development of revolutionary Math discoveries on the ground that entities like imaginary numbers "do not really exist".

Historical Argument

Since its birth at Pythagorean schools, Math was considered as a primary reality, as a set of perfect divine ideas.

As such, it was introduced into Platonic ontology, receiving its high primary value.

Great mathematicians of antiquity were all platonics, they developed Math for its own sake, as a special religious art and a special communion with God.

The same view on Math shared Galileo, Kepler and Descartes. The perfection of Math dictated for them the belief that "the book of nature is written in language of mathematics".

Thus, high ontological status of Math, entangled with its special eternal esthetics, was crucial for its birth and development, as well as for the birth of Physics.

Birth of Mathematics (Jean Dieudonne)

1906–1992

While all ancient civilisations, in order to satisfy the needs of daily life, had to develop procedures of arithmetical calculation and spatial measurement, only the Greeks, from the sixth century B.C., thought of analysing the chain of reasoning behind these procedures, and thus created an entirely new mode of thinking… In §§2 and 3 we shall focus on the two fundamental characteristics of Greek mathematics:

1) The idea of *proof*, by a succession of logical inferences from unproved propositions, axioms and postulates. It must be emphasized that this idea could only be brought into play thanks to the skill acquired in the manipulation of logic by those nursed in the Greek schools of philosophy. A particularly striking example is supplied by the principle of "proof by contradiction", a tool refined by logicians which became one of the pillars of mathematical reasoning.

2) The objects with which mathematicians are concerned have the same names as those which are used in practical calculations: numbers, geometrical figures and magnitudes. But, from the time of Plato, mathematicians have been aware that under these names they are reasoning about entirely different entities, *immaterial* entities, obtained "by abstraction" from objects which are accessible to our senses, but which are only "pictures" of the former.

We show in §4, which is about diagrams in geometry, how the properties attributed by the axioms t:o the "abstract" objects of geometry make them profoundly different from their "pictures", and what difficulties arise because of this in finding an appropriate vocabulary to *define* these objects.

Platonic Mysticism

427–347

Theaetetus 176

THEODORUS: Socrates, if you persuade the whole world as well as me, there would be more peace and less evil among men.

SOCRATES: But it is not possible that evil should ever disappear, Theodorus. For it is necessary that there always be something that is more or less contrary to the good (ὑπεναντίον). And this thing cannot have its place among the gods; but it is necessary that it circulate in the realm of mortal nature, in this world. This is why it is necessary to be forced to flee from here below as fast as one can. This flight from the world is *assimilation* to God in the measure to which that is possible.

This assimilation consists in becoming just and holy with the help of reason. But, my dear man, it is not easy to persuade men that it is necessary to flee from sin and to seek virtue for any other motive than the one that is common to human beings, who do not want to appear to be bad people, who want to appear virtuous. That is the foolishness of an old woman, I believe. The right reason is this one: *Never, in any way, is God unjust*. He is just to the supreme degree, and there is nothing more like him than the one who among us is the most just. Knowing that is wisdom and true virtue. To be ignorant of it is to be manifestly stupid and vile. The other apparent habits, the other forms of wisdom that deal with politics, power, technique, are crude and mercenary. With respect to

The last counter-argument: Skepticism

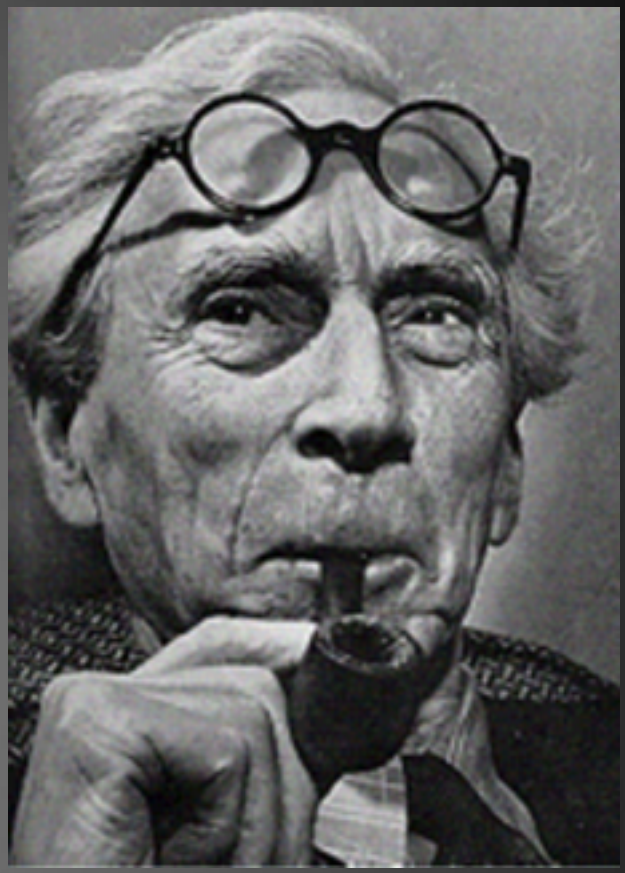
History shows that philosophical problems are unsolvable on a rational ground; only scientific approach is reliable to some degree. Future science will show how wrong we were today. We should not overestimate our modest abilities and concentrate on the well established scientific problems. All philosophical arguments are unreliable, and it is a waste of time to discuss them and think about them.

There is certain truth in this position. However, without its dialectic counter-balance, this truth turns to be a fallacy, revealing itself in its fruitlessness. Among founding fathers of mathematics or physics, there were no skeptics. Many great mathematicians expressed belief in the platonic world, and none, as far as I know, fully denied it.

Bertrand Russell on Scepticism

Scepticism naturally made an appeal to many unphilosophic minds. People observed the diversity of schools and the acerbity of their disputes, and decided that all alike were pretending to knowledge which was in fact unattainable. Scepticism was a lazy man's consolation, since it showed the ignorant to be as wise as the reputed men of learning. To men who, by temperament, required a gospel, it might seem unsatisfying, but like every doctrine of the Hellenistic period it recommended itself as an antidote to worry... For these reasons, Scepticism enjoyed a considerable popular success...

It should be observed that Scepticism as a philosophy is not merely doubt, but what may be called dogmatic doubt... The philosophical Sceptic says 'Nobody knows, and nobody ever can know.' It is this element of dogmatism that makes the system vulnerable. Sceptics, of course, deny that they assert the impossibility of knowledge dogmatically, but their denials are not very convincing.



1872–1970

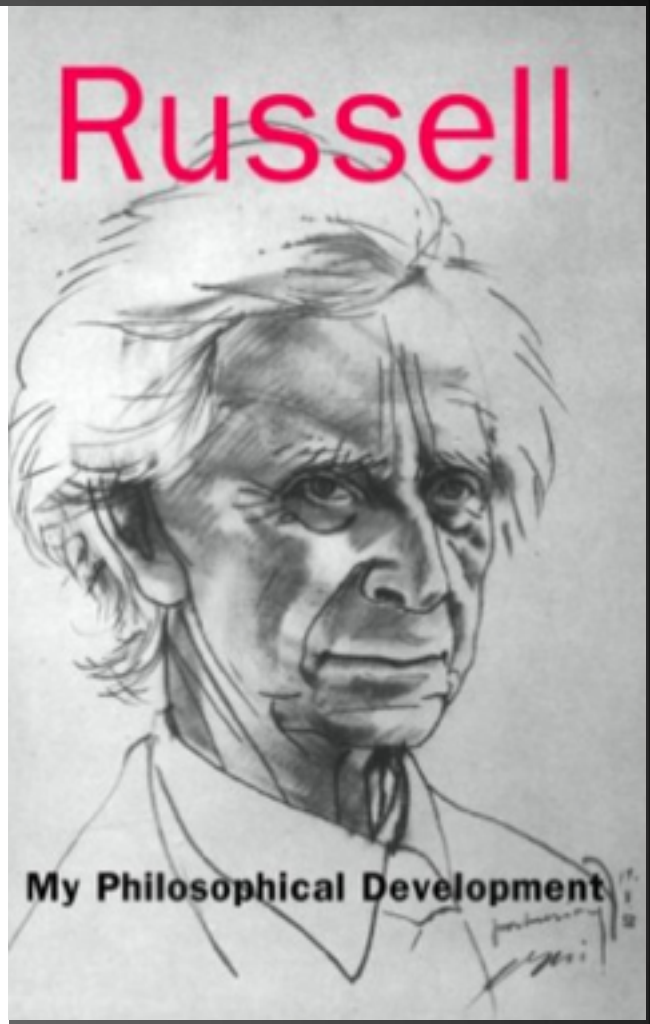
History of Western Philosophy (1946)

Russell's 'Retreat from Pythagoras'

fying. But this mood began to pass, and was finally dispelled by the First World War.

One effect of that War was to make it impossible for me to go on living in a world of abstraction. I used to watch young men embarking in troop trains to be slaughtered on the Somme because generals were stupid. I felt an aching compassion for these young men, and found myself united to the actual world in a strange marriage of pain. All the high-flown thoughts that I had had about the abstract world of ideas seemed to me thin and rather trivial in view of the vast suffering that surrounded me. The non-human world remained as an occasional refuge, but not as a country in which to build one's permanent habitation.

In this change of mood, something was lost, though something also was gained. What was lost was the hope of finding perfection and finality and certainty. What was gained was a new submission to some truths which were to me repugnant. My abandonment of former beliefs was, however, never complete. Some things remained with me, and still remain: I still think that truth depends upon a relation to fact, and that facts in general are non-human; I still think that man is cosmically unimportant, and that a Being, if there were one, who could view the universe impartially, without the bias of *here* and *now*, would hardly mention man, except perhaps in a footnote near the end of the volume; but I no longer have the wish to thrust out human elements from regions where they belong; I have no longer the feeling that intellect is superior to sense, and that only Plato's world of ideas gives access to the 'real' world. I used to think of sense, and of thought which is built on



1959

Further Considerations on the Terminus

If Math World is real, where does it exist?

A speculative entity may exist only in some mind. Furthermore, keeping the name '*mind*' or, better, Mind, in that specific case, the same word as for our minds, we would stress human ability to see the content of that entity where the Math forms are, which is important.

The Mind in that respect is atemporal; thus, it cannot be the Creator, because the act of creation implies time or a birth of time, which is completely foreign for Math.

Thus, either time is nothing but our illusion, and we are like characters of eternal novels, or God is paradoxical, He is both outside and inside time.

This dilemma is both important and unsolvable for the objective reasoning. I am solving it on the ethical ground:

Since only the latter solution, of the paradoxical God, is compatible with the meaning of life, duty, cognition and creativity, I am freely accepting it.

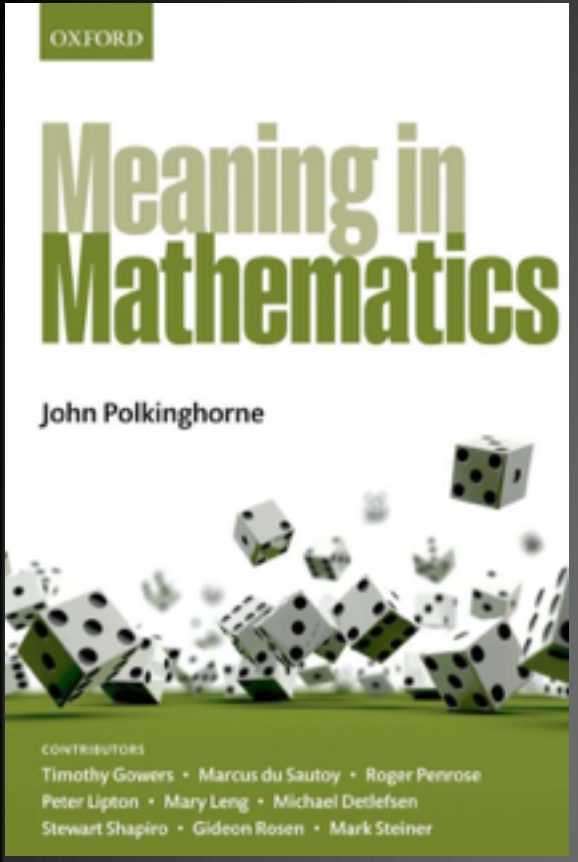
Conclusion

Reality of the Math Platonic World is a metaphysical question; it cannot be solved in a scientific manner.

There are strong arguments for Math realism: logical, operational, historical, ethical, esthetic.

Math realism is denied by naturalism and is disregarded by skepticism. What are the values of naturalism and skepticism though?

I suggested the arguments; it is up to each of you to decide which of them matter more.


OXFORD
Meaning in Mathematics
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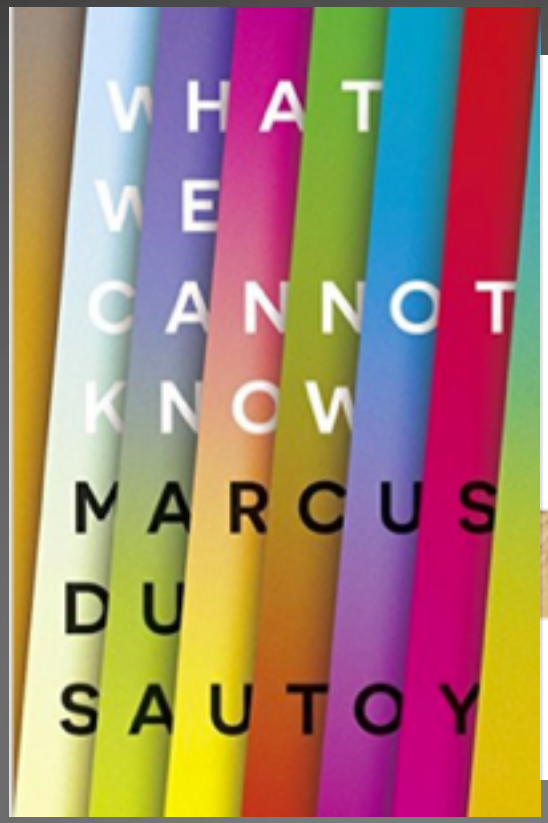
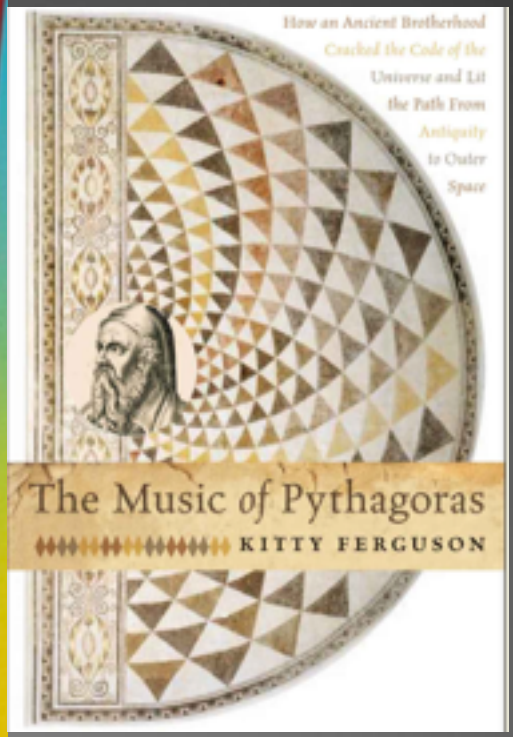

WHAT WE CANNOT KNOW
MARCUS DU SAUTOY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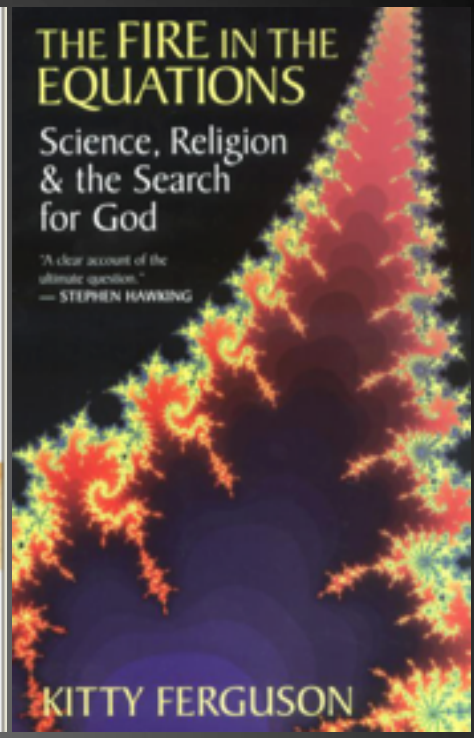
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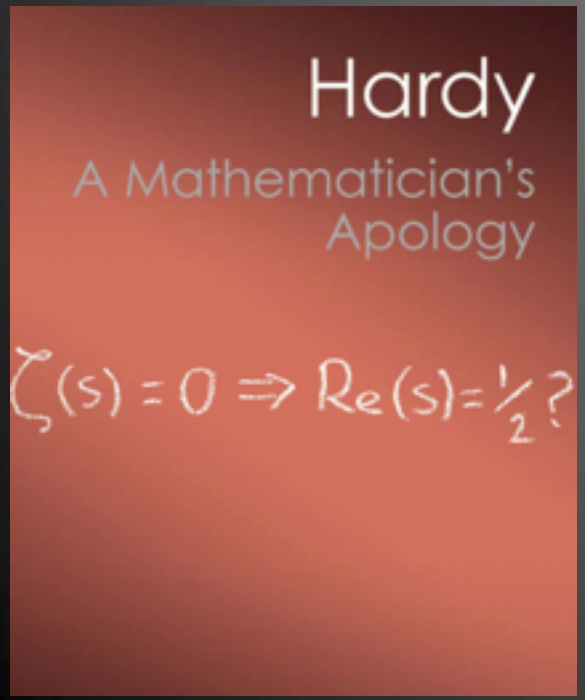
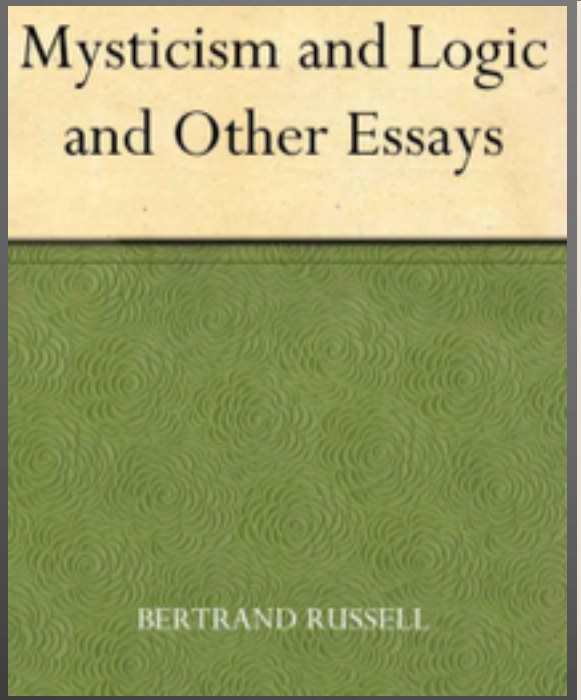

Hardy
A Mathematician's Apology
ζ(s) = 0 ⇒ Re(s) = ½ ?


Mysticism and Logic and Other Essays
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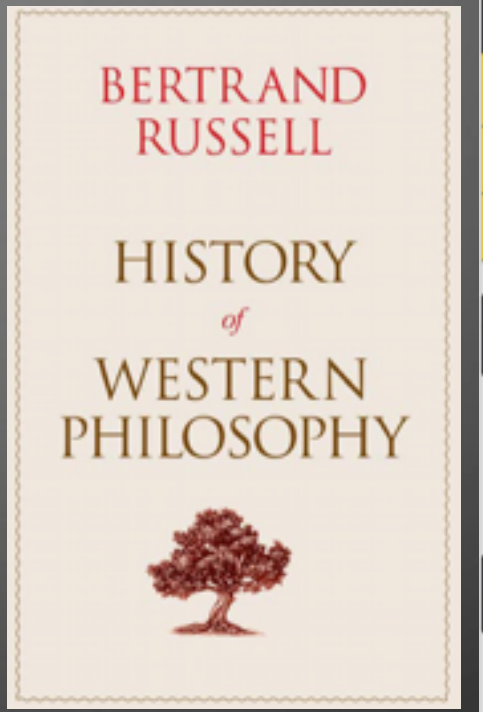
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